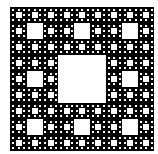
Non-Smooth Boundary Value Problems

Alexander Teplyaev

joint research with

Anna Rozanova-Pierrat, Gabriel Claret (Paris-Saclay),
Michael Hinz (Bielefeld), Marco Carfagnini (Melbourne),
Masha Gordina, Luke Rogers (UConn), et al.



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Abstract:

- The lectures will begin with a review of the general functional analysis framework, covering the **Hille–Yosida theorem**, the spectral theory of self-adjoint operators as developed by von Neumann, the theory of positive quadratic forms, **Dirichlet forms** and Markov operators by Beurling and Deny, and the related theory of symmetric Markov stochastic processes (Kolmogorov, Levy, Doob, Hunt, Dynkin).
- Next, I will discuss applications of the theory of **ultra-contractive semigroups**, based on recent joint work with Carfagnini and Gordina, following the work of E.B. Davis.
- After that, I will present recent progress in non-smooth **Dirichlet, Neumann, and Robin Boundary Value Problems**, which are the result of joint work with Hinz, Magoulès, and Rozanova-Pierrat.
- Another application of the general theory will deal with non-smooth **Wentzell Boundary Value Problems**, a joint work with Hinz, Lancia, and Vernole.
- If time permits, I will also discuss recent advancements in **non-smooth layer potentials and Riemann-Hilbert problems** in a joint work with Claret and Rozanova-Pierrat.

Lectures 3 and 4 – Geometric Analysis and Applications

1. Non-Smooth Dirichlet, Neumann, and Robin Boundary Value Problems

2. Relative compactness of of non-smooth Uniform Domains and fractal boundary measures

3. Non-Smooth Wentzell Fractal Snowflake Problems

4. Layer Potentials and convergence

5. Applications of these ideas to Machine Learning

1. Non-Smooth Dirichlet, Neumann, and Robin Boundary Value Problems

- Michael Hinz, Anna Rozanova-Pierrat, Alexander Teplyaev **Non-Lipschitz uniform domain shape optimization in linear acoustics**, SIAM J. Control Optim. 59 (2021), no. 2, 1007–1032. doi.org/10.1137/20M1361687 (SICON) *arXiv:2008.10222*

- Michael Hinz, Anna Rozanova-Pierrat, Alexander Teplyaev, **Boundary value problems on non-Lipschitz uniform domains: Stability, compactness and the existence of optimal shapes** DOI: 10.3233/ASY-231825 Asymptotic Analysis, 134 (2023) 25–61 *arXiv:2111.01280*

2. Relative compactness of of non-smooth Uniform Domains and fractal boundary measures

- Michael Hinz, Anna Rozanova-Pierrat, Alexander Teplyaev **Non-Lipschitz uniform domain shape optimization in linear acoustics**, SIAM J. Control Optim. 59 (2021), no. 2, 1007–1032. doi.org/10.1137/20M1361687 (SICON) *arXiv:2008.10222*

- Michael Hinz, Anna Rozanova-Pierrat, Alexander Teplyaev, **Boundary value problems on non-Lipschitz uniform domains: Stability, compactness and the existence of optimal shapes**, DOI: 10.3233/ASY-231825 Asymptotic Analysis, 134 (2023) 25–61 *arXiv:2111.01280*

Banach–Alaoglu theorem:

The closed unit ball of the dual space of a normed vector space is compact in the weak* topology

- Bounded closed sets in a Hilbert space are weakly compact
- Probability measures are weakly compact on a “nice” compact metric space
- Uniformly bounded closed sets of linear operators on a Hilbert space are compact under strong operator convergence
- Closed sets of contraction semi-groups are compact under the strong operator convergence
- Closed sets of (even unbounded) coercive operators are compact under the strong resolvent convergence

3. Non-Smooth Wentzell Fractal Snowflake Problems

- Michael Hinz, Maria Rosaria Lancia, Alexander Teplyaev, Paola Vernole, **Fractal snowflake domain diffusion with boundary and interior drifts**, J. Math. Anal. Appl. 457 (2018), no. 1, 672–693. https://doi.org/10.1016/j.jmaa.2017.07.065 *arXiv:1605.06785*

- Malcolm Gabbard, Carlos Lima, Gamal Mograby, Luke G. Rogers, Alexander Teplyaev **Discretization of the Koch Snowflake Domain with Boundary and Interior Energies**, SEMA SIMAI Springer Series ICIAM2019 Fractals in engineering: Theoretical aspects and Numerical approximations (2021) Pages 79-102 DOI 10.1007/978-3-030-61803-2 *arXiv:2002.04680*

4. Layer Potentials and convergence

- Gabriel Claret, Michael Hinz, Anna Rozanova-Pierrat, Alexander Teplyaev, **Layer potential operators for transmission problems on extension domains**, *arXiv:2403.11601*

- Gabriel Claret, Anna Rozanova-Pierrat, Alexander Teplyaev, **Convergence of Layer Potentials and Riemann-Hilbert Problem on Extension Domains**, tba

5. Applications of these ideas to Machine Learning

- Mikhail Belkin and Partha Niyogi. **Laplacian eigenmaps for dimensionality reduction and data representation**. Neural computation, 15(6):1373–1396, 2003

- Bernard Akwei, Bobita Atkins, Rachel Bailey, Ashka Dalal, Natalie Dinin, Jonathan Kerby-White, Tess McGuinness, Tonya Patricks, Luke Rogers, Genevieve Romanelli, Yiheng Su, Alexander Teplyaev, **Convergence, optimization and stability of singular eigenmaps**, *arXiv:2406.19510*

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    - *If time permits: Semi-Supervised Machine Learning*