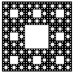
Convergence of eigenvalues and diffusions in non-smooth settings

Alexander Teplyaev

joint research with

Michael Hinz (Bielefeld), Masha Gordina, Luke Rogers (UConn), Marco Carfagnini (UCSD), Anna Rozanova-Pierrat, Gabriel Claret (Paris-Saclay) et al.



FGS7 Chemnitz September 2024

Abstract:

Dirichlet form analysis provides powerful tools for studying diffusions and spectral analysis in non-smooth settings, with Mosco convergence being a standard approach for examining approximations. However, Mosco convergence alone may not suffice to understand finer properties, such as the convergence of eigenvalues and small deviations of diffusion processes. This talk will present recent results that strengthen the Mosco convergence of Dirichlet forms in non-smooth spaces, including fractals, domains with fractal boundaries, and sub-Riemannian spaces. The presentation includes joint work with Michael Hinz, Anna Rozanova-Pierrat, Gabriel Claret, Luke Rogers, Marco Carfagnini, and Masha Gordina.

Plan of the talk:

- Mosco convergence, strong and norm resolvent convergence
- Introduction and motivation, analysis on “fractafolds”*
- Wave absorption: numerical shape optimization
- Wave absorption: theoretical shape optimization
- Equations used in architecture
- Wentzell Boundary conditions
  - Theoretical study
  - Discrete approximations
- 1. Convergence of eigenvalues in fractal domains
- 2. Discrete spectrum for Dirichlet forms
- 3. Small deviations
- 5. Convergence of the Dirichlet heat kernels
- 6. Local convergence of stochastic flows
- New Frontiers: Layer potentials
  - Riemann-Hilbert and Poincare variational problems
  - Hilbert transform
  - Maxwell and other vector equations

Mosco convergence, strong and norm resolvent convergence

- Mosco, Umberto *Convergence of convex sets and of solutions of variational inequalities*. Advances in Math. 3 (1969), 510–585.
- Mosco, Umberto *Composite media and asymptotic Dirichlet forms*. J. Funct. Anal. 123 (1994), no. 2, 368–421.

Kato, Tosio
Perturbation theory for linear operators. Springer-Verlag 1966.

[Reed-Simon 1972]: For non-negative closed quadratic forms,

- **Mosco convergence is equivalent to the strong resolvent convergence**,
- but is **weaker than the norm resolvent convergence**.

Mosco convergence does not imply convergence of the spectrum

$\operatorname*{M\text{-}lim}_{n\to\infty} E_n = F$ or $E_n \xrightarrow[n\to\infty]{\mathrm{M}} F$.

- $x_n \in L^2$ converging weakly to $x \in L^2$, $\liminf_{n\to\infty} E_n(x_n) \geq F(x)$;
- for each $x \in L^2$ there exists an approximating sequence of elements $x_n \in L^2$, converging strongly to x, such that $\limsup_{n\to\infty} E_n(x_n) \leq F(x)$.

Example:

$L^2 := \ell^2(\mathbb{Z}_+)$

$E_n((x_k)) := \sum_{k\geqslant n} |x_k|^2 \xrightarrow[n\to\infty]{\mathrm{M}} E = 0$

$\sigma(E_n) = \{0, 1\} \neq \{0\} = \sigma(E)$

Introduction and motivation, analysis on “fractafolds”*

- *Strichartz: *A fractafold, a space that is locally modeled on a specified fractal, is the fractal equivalent of a manifold.*
  - *A “fractafold” is to a fractal what a manifold is to a Euclidean half-space.*

This is a part of the broader program to develop probabilistic, spectral and vector analysis on singular spaces by carefully building approximations by graphs or manifolds.

What is the first appearance of fractals is science?

In a sense, the simplest possible fractal appears in the famous Zeno's paradoxes: Zeno of Elea (c. 495 – c. 430 BC) "Achilles and the Tortoise"

1. Achilles runs to the tortoise's starting point while the tortoise walks forward.
2. Achilles advances to where the tortoise was at the end of Step 1 while the tortoise goes yet further.
3. Achilles advances to where the tortoise was at the end of Step 2 while the tortoise goes yet further.
   Etc.

Apparently, Achilles never overtakes the tortoise, since however many steps he completes, the tortoise remains ahead of him.

Dichotomy paradox: that which is in locomotion must arrive at the half-way stage before it arrives at the goal. In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead. [Aristotle, Physics VI:9, 239b10, 239b15]

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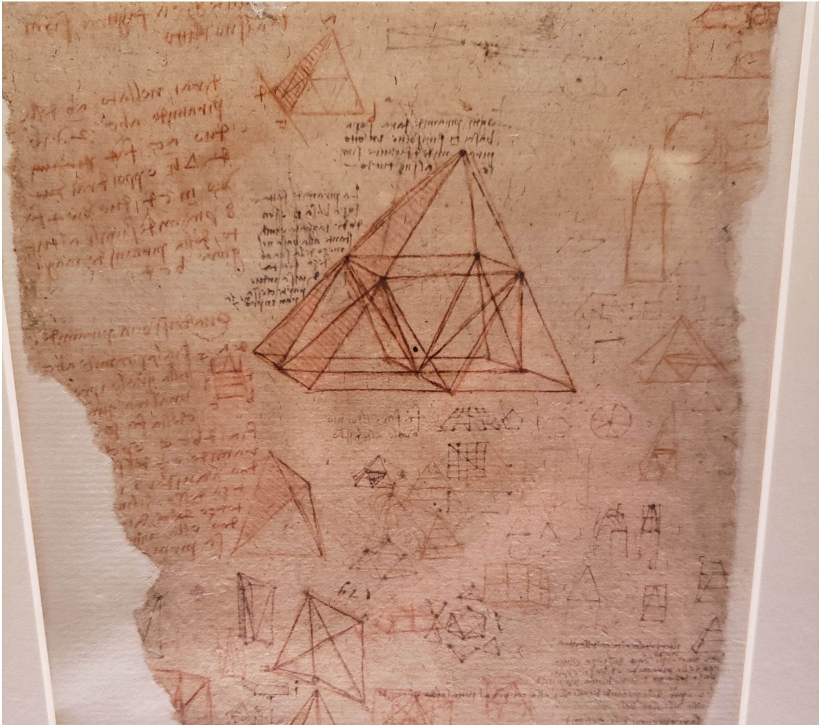
In 1821, Augustin-Louis Cauchy proved that, for $-1 < x < 1$,

$$a + ax + ax^2 + ax^3 + ... = \frac{a}{1-x} := S(a, x)$$

This is a weakly-self-similar sum satisfying a re-normalization "fixed-point" functional equation

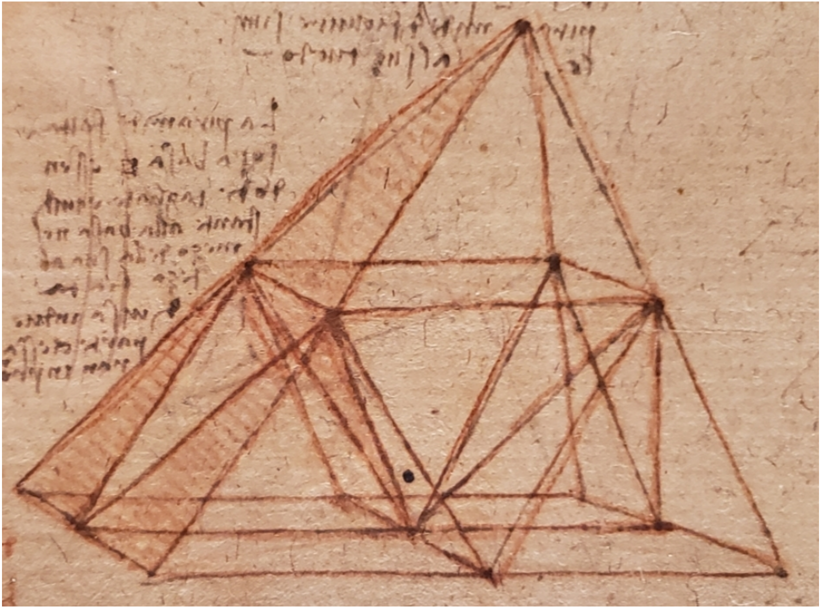
$$S(a, x) = a + x \cdot S(a, x)$$



MATHEMATICS AND GEOMETRY: ***Decomposition of pyramids***

Red pencil, pen and ink, c. 1515

The sheet shows several diagrams of pyramids broken down into smaller ones. The caption above the major pyramid drawing states that each pyramid with a square base "is resolved into eight pyramids of figures similar to its whole." The same concept is reiterated by the smaller diagram above the larger pyramid. Below, there is another small sketch with a caption explaining how to square a pyramid. Under the base of the main pyramid there is a note that alludes to a German craftsman, while immediately to the side there are grids that could be exercises in perspective.



Cantor, Sierpinski, Julia, Mandelbrot

- How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension (Mandelbrot 1967).

The coastline paradox: the measured length of a stretch of coastline depends on the scale of measurement.

Fractal titanium oxide under inverse 10-ns laser deposition in air and water. A. Pan, W. Wang, X. Mei, Q. Lin, J. Cui, K. Wang, Z. Zhai Applied Physics A volume 123, Article number: 253 (2017)

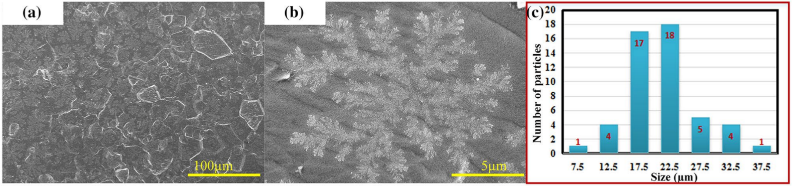
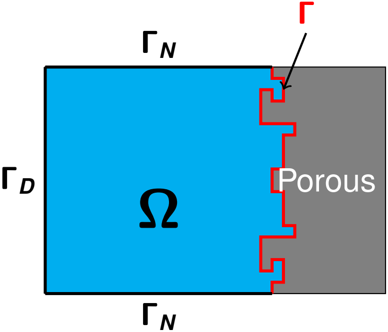
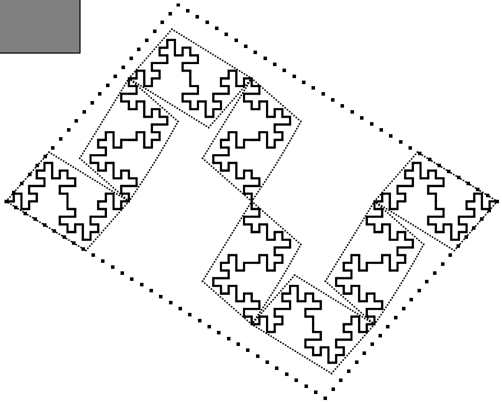


Fig. 5 Surface morphology of titanium with the laser energy of 86 mJ, scanning speed of 0.01 mm/s, and scan length of 10 mm. *Inset* **a** depicts the surface morphology beyond laser irradiation zone. *Inset* **b** shows a typical fractal structure unit, and *inset* **c** is size distribution histograms of 50 fractal structure units

Wave absorption: numerical shape optimization

- F. Magoulès, T.P. Kieu Nguyen, P. Omnes, A. Rozanova-Pierrat,
  Optimal absorption of acoustic waves by a boundary.
  SIAM J. Control Optimization 59 (2021)
  + more numerical results
- C. Bardos, D. Grebenkov, A. Rozanova-Pierrat,
  Short-time heat diffusion in compact domains with discontinuous transmission boundary conditions.
  Math. Mod. Meth. Appl. Sci. 26 (2016)
- A. Rozanova-Pierrat, D. S. Grebenkov, and B. Sapoval,
  Faster diffusion across an irregular boundary.
  Phys. Rev. Lett. 108 (2012)


Γ
Γ_N
Γ_D
Ω
Porous
Γ_N



Wave absorption: theoretical shape optimization

- M. Hinz, A. Rozanova-Pierrat, A. Teplyaev, *Non-Lipschitz uniform domain shape optimization in linear acoustics*. SIAM J. Control Optim. 59 (2021)

- M. Hinz, A. Rozanova-Pierrat, A. Teplyaev, *Boundary value problems on non-Lipschitz uniform domains: Stability, compactness and the existence of optimal shapes*. Asymptotic Analysis (2023)

Equations used in architecture

- M. Hinz, F. Magoulès, A. Rozanova-Pierrat, M. Rynkovskaya, A. Teplyaev, *On the existence of optimal shapes in architecture*. Applied Mathematical Modelling 94 (2021)

Given a domain $\Omega \subset \mathbb{R}^N$ and a vector field $\mathbf{v} \in W^{1,2}(\Omega)^N$ we denote the symmetric part of its gradient by

$$e(\mathbf{v}) = \frac{1}{2}\left(\nabla \mathbf{v} + (\nabla \mathbf{v})^t\right).$$

Let $A \in L^\infty(\Omega, \mathcal{M}_N^s(\alpha, \beta))$ and write $\sigma(\mathbf{v}) = Ae(\mathbf{v})$, $\mathbf{v} \in W^{1,2}(\Omega)^N$. We are interested in solutions $\mathbf{u} \in W^{1,2}(\Omega)^N$ of BVP:

$$\begin{cases} -\mathbf{div}\,\sigma(\mathbf{u}) &= \mathbf{f} \quad \text{in } \Omega, \\ \mathbf{u} &= \mathbf{0} \quad \text{on } \Gamma_{\mathbf{Dir}}, \\ \sigma(\mathbf{u}) \cdot n &= \mathbf{g} \quad \text{on } \Gamma_{\mathbf{Neu}}. \end{cases} \tag{1}$$

Wentzell Boundary conditions

- A. Wentzell. On boundary conditions for multi-dimensional diffusion processes. Theor. Probability Appl. (1959)

$$\mathcal{E}(u) = \int_{\Omega} \|\nabla u\|^2 dx + \mathcal{E}_{\partial\Omega}(u)$$

Theoretical study

- M. R. Lancia, P. Vernole,
  Venttsel' problems in fractal domains
  J. Evol. Equ. 14 (2014), no. 3, 681–712.

  ...

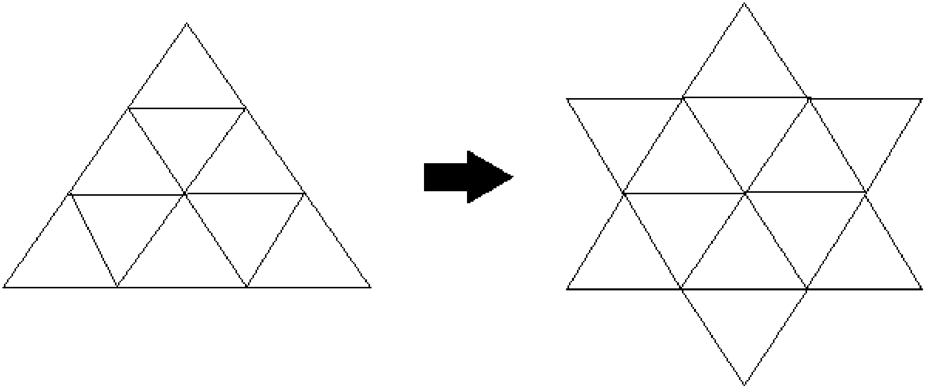
- M. Hinz, M. R. Lancia, A. Teplyaev, P. Vernole, Fractal snowflake domain diffusion with boundary and interior drifts, J. Math. Anal. Appl. 457 (2018)

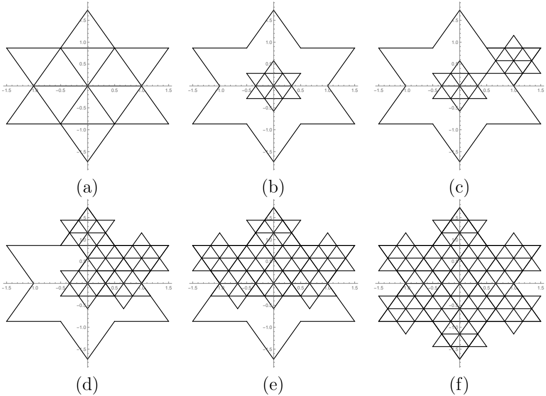
$$\mathcal{E}(u) = \int_{\Omega} \|\nabla u\|^2 dx + \mathcal{E}_{\partial\Omega}(u)$$

Discrete approximations

- M. Gabbard, C. Lima, G. Mograby, L. G. Rogers, A. Teplyaev, Discretization of the Koch Snowflake Domain with Boundary and Interior Energies, SEMA SIMAI Springer Series ICIAM2019 Fractals in engineering: Theoretical aspects and Numerical approximations (2021)

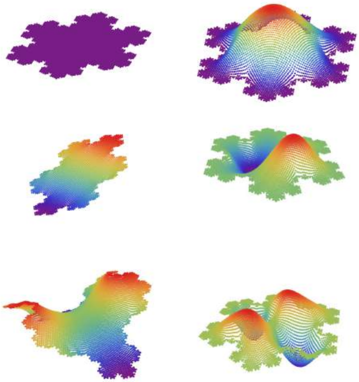
$$\mathcal{E}(u) = \int_{\Omega} \|\nabla u\|^2 dx + \mathcal{E}_{\partial\Omega}(u)$$

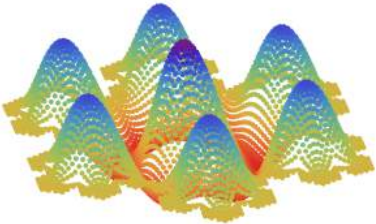


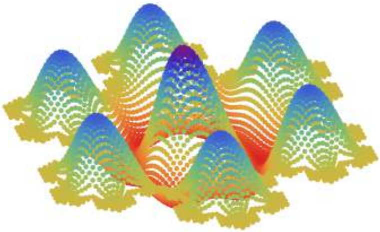


(a) (b) (c)

(d) (e) (f)







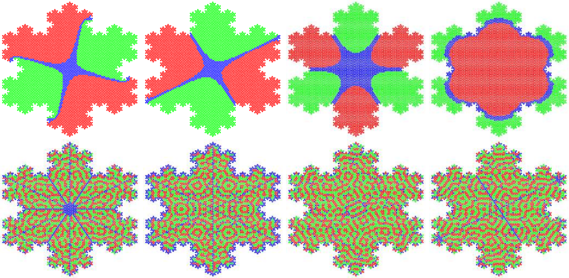


FIGURE 5. Contour Plots of the Eigenvectors of L_n corresponding to eigenvalues λ: (a) 4th eigenvector, $\lambda = 48.1$. (b) 5th eigenvector, $\lambda = 48.1$. (c) 6th eigenvector, $\lambda = 85.1$. (d) 8th eigenvector $\lambda = 125.4$. (e) 1153rd eigenvector $\lambda = 49965.7$. (f) 1157th eigenvector $\lambda = 50156.6$. (g) 1161st eigenvector, $\lambda = 50188.8$ and (h) 1162nd eigenvector, $\lambda = 50188.83$. Blue regions indicate the values of an eigenvector in $(-\epsilon, \epsilon)$, red regions in (ϵ, ∞) and green regions in $(-\infty, -\epsilon)$, where $\epsilon = 0.01$. (Level 4 graph approximation)

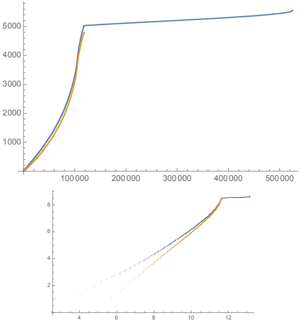


FIGURE 6. (Upper) Eigenvalue counting functions of Dirichlet Laplacian (orange) and L_n (blue). (Lower) Log-Log plot of the eigenvalue counting functions of Dirichlet Laplacian (orange) and L_n (blue) (Level 4 graph approximation).

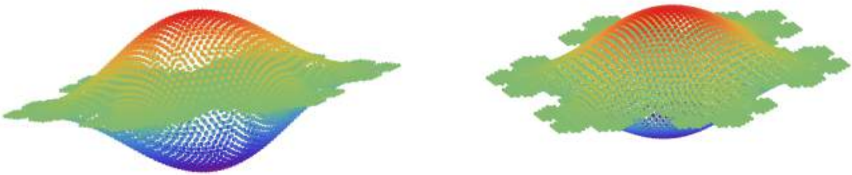


FIGURE 7. (a) The $5,028$th eigenvector of L_n, $\lambda = 118038.02$. (b) The last Dirichlet eigenvector, $\lambda = 118039.37$. The oval-shaped graph is due to a high oscillation of both eigenvectors

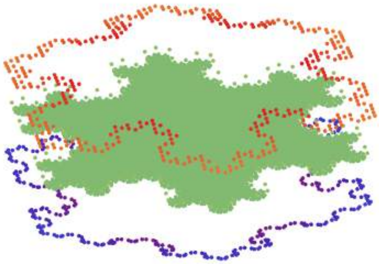


FIGURE 8. The last L_n eigenvector, $\lambda = 524999.69$. The graph splits into two parts, above and below the Koch snowflake domain due to a high oscillation (Level 4 graph approximation).

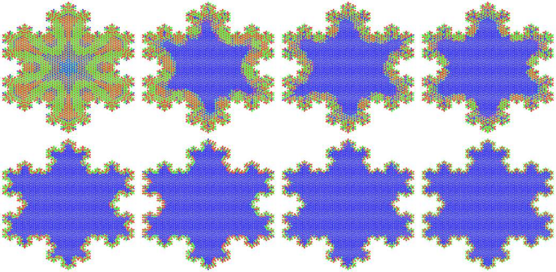
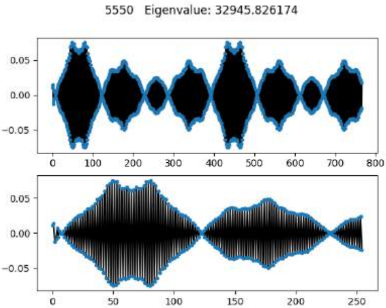


FIGURE 9. L_n eigenvectors localization with eigenvalues λ: (a) 5030th eigenvector, $\lambda = 118048.66$. (b) 5031th eigenvector, $\lambda = 119678.65$. (c) 5032th eigenvector, $\lambda = 119678.65$. (d) 5033th eigenvector, $\lambda = 121460.72$. (e) 5100th eigenvector, $\lambda = 185367.41$. (f) 5200th eigenvector, $\lambda = 291364.38$. (g) 5300th eigenvector, $\lambda = 392584.97$. (h) 5557th eigenvector, $\lambda = 524999.69$. Blue regions indicate the values of an eigenvector in $(-\epsilon, \epsilon)$, red regions in (ϵ, ∞) and green regions in $(-\infty, -\epsilon)$, where $\epsilon = 0.01$ (Level 4 graph approximation).


5550 Eigenvalue: 32945.826174
0.05
0.00
−0.05
0
100
200
300
400
500
600
700
800
0.05
0.00
−0.05
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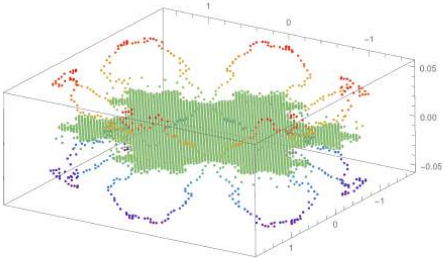
50
100
150
200
250


1
0
-1
0.05
0.00
-0.05
-1
0
1

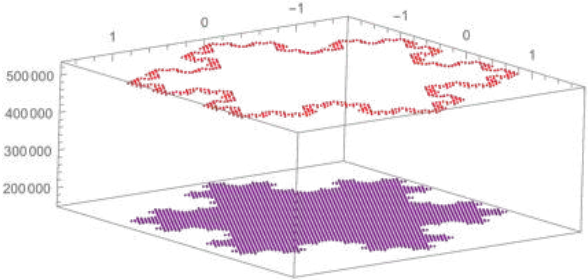


FIGURE 10. The high frequency landscape vector attains just the following two values the boundary vertices 527360 and 524288. It is constant on the interior vertices with the value 157464. (Level 4 graph approximation)

1. Convergence of eigenvalues in fractal domains

Theorem (Hinz, Rozanova-Pierrat, T.)

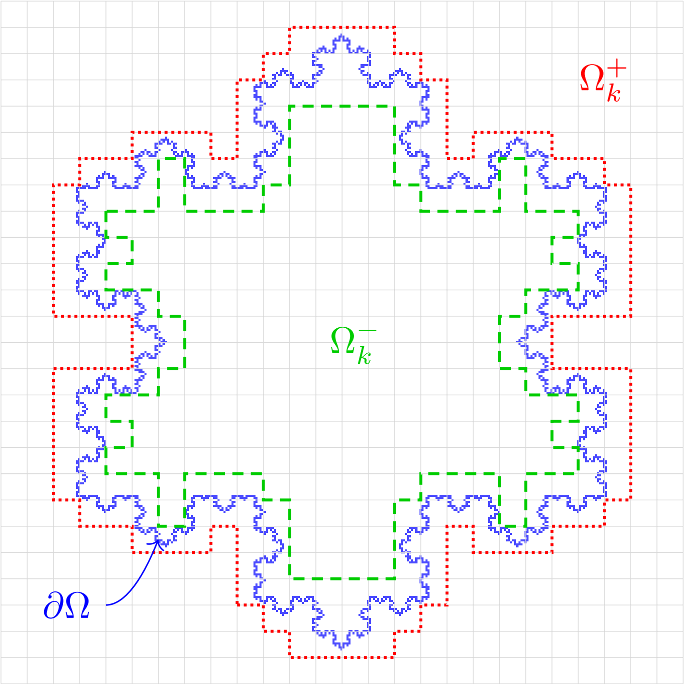
Let n, D, α, γ, ε, d, $(\Omega_m)_m$ *and* $(\mu_m)_m$ *be a* **a sequence of admissible domains**. *Suppose that* $\lim_m \Omega_m = \Omega$ *in the Hausdorff sense and in the sense of characteristic functions and* $\lim_m \mu_m = \mu$ *weakly. There is a sequence* $(m_k)_{k=1}^{\infty}$ *with* $m_k \uparrow +\infty$ *such that the following hold.*

(i) *We have* $\lim_{k\to\infty} P_{\Omega_{m_k}} \circ \hat{G}_{\alpha,\gamma}^{\Omega_{m_k},\mu_{m_k},*} = P_\Omega \circ \hat{G}_{\alpha,\gamma}^{\Omega,\mu,*}$ *in operator norm.*

(ii) *If* $0 < a < b$ *are in the resolvent set of* $-\mathcal{L}_\gamma^{\Omega,\mu,*}$, *then* $\lim_{k\to\infty} \pi_{(a,b)}(\Omega_{m_k}, \mu_{m_k}, *) = \pi_{(a,b)}(\Omega, \mu, *)$ *in operator norm.*

(iii) *The spectra of the operators* $-\mathcal{L}_\gamma^{\Omega_{m_k},\mu_{m_k},*}$ *converge to the spectrum of* $-\mathcal{L}_\gamma^{\Omega,\mu,*}$ *in the Hausdorff sense. The eigenvalues* $\lambda_n(\Omega,\mu,*)$ *of the operator* $-\mathcal{L}_\gamma^{\Omega,\mu,*}$ *are exactly the limits as* $k \to \infty$ *of sequences of the eigenvalues of the operators* $-\mathcal{L}_\gamma^{\Omega_{m_k},\mu_{m_k},*}$,

$$\lambda_n(\Omega,\mu,*) = \lim_{k\to\infty} \lambda_n(\Omega_{m_k}, \mu_{m_k}, *). \qquad (2)$$


Ω_k^+
Ω_k^-
$\partial\Omega$

2. Discrete spectrum for Dirichlet forms

Theorem (Carfagnini, Gordina, T.)

Let $\mathcal{U}$ be an open bounded subset of $\mathcal{X}$, and $P_t^{\mathcal{U}}$ be the semigroup associated to $(\mathcal{E}, \mathcal{D}_{\mathcal{E}})$ with the infinitesimal generator $A_{\mathcal{U}}$. Assume that $p_t(x, y)$ exists for all t and for m-a.e. $x, y \in \mathcal{X}$. If there exists a $t_{\mathcal{U}} > 0$ such that

$$M_{\mathcal{U}}(t_{\mathcal{U}}) = \operatorname*{ess\,sup}_{(x,y)\in\mathcal{U}\times\mathcal{U}} p_{t_{\mathcal{U}}}(x, y) < \frac{1}{m(\mathcal{U})^2}, \tag{3}$$

then the spectrum of $-A_{\mathcal{U}}$ is discrete and $\lambda_1 > 0$.

3. Small deviations

Theorem (Carfagnini, Gordina, T.)

Let $\{P_t\}_{t\geqslant 0}$ be a strongly continuous contraction semigroup on $L^2(\mathcal{X}, m)$. Let $x \in \mathcal{X}$ and assume that $P_t^{B_1(x)}$ is irreducible. Assume that the heat kernel $p_t(x, y)$ exists for all t and for all $x, y \in \mathcal{X}$ and that

$$p_t(x, y) \leqslant c\, t^{-\frac{\alpha}{\beta}}$$

for any t, x, and y. Moreover, assume that there exists a t_0 such that $p_{t_0}(x, y)$ is continuous for $x, y \in \mathcal{X}$. If X_t^x is **self-similar** *then*

$$\lim_{\varepsilon \to 0} e^{\lambda_1 \frac{t}{\varepsilon^\beta}} \mathbb{P}^x \left(\sup_{0 \leqslant s \leqslant t} d(X_s, x) < \varepsilon \right) = c_1 \varphi_1(x), \tag{4}$$

where $\lambda_1 > 0$ is the spectral gap of A restricted to the unit ball $B_1(x)$, and φ_1 is the corresponding positive eigenfunction.

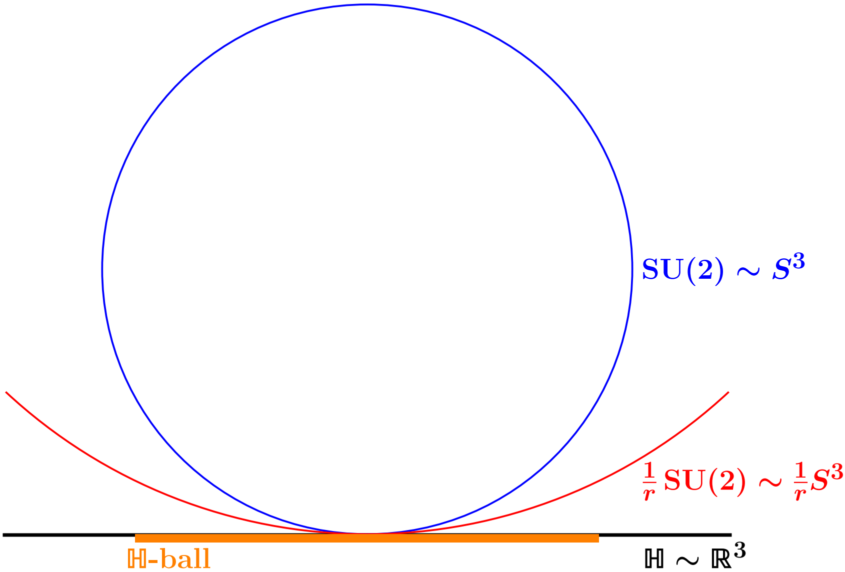
4. Convergence of the re-normalized eigenvalues of small balls in $\mathbf{SU(2)}$ to corresponding eigenvalues in the unit ball of $\mathbb{H}$

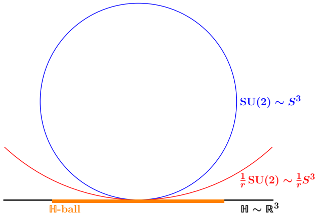
Here $\mathbb{H}$ is the Heisenberg group, which is a re-scaled limit of $\mathbf{SU(2)}$ near the identity.

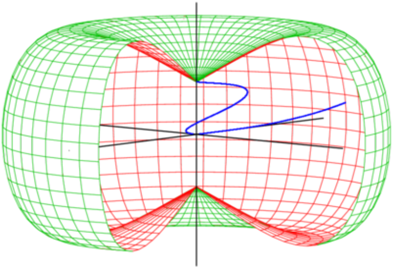
Theorem (Carfagnini, Gordina, T.)

Let $0 < \lambda_1^{\mathbb{H}} < \lambda_2^{\mathbb{H}} \leqslant \lambda_3^{\mathbb{H}} \leqslant \dots$ be the Dirichlet eigenvalues in the unit ball $B_1^{\mathbb{H}}$ of $\mathbb{H}$, counted with multiplicity. Let $0 < \lambda_1^r < \lambda_2^r \leqslant \lambda_3^r \leqslant \dots$ be the Dirichlet eigenvalues in the r-ball $B_r^{\mathbf{SU(2)}}$ of $\mathbf{SU(2)}$, counted with multiplicity. Then for each $n \geqslant 1$ we have

$$\lim_{r \to 0} r^2 \lambda_n^r = \lambda_n^{\mathbb{H}}. \tag{5}$$


$\mathbf{SU(2)} \sim S^3$
$\frac{1}{r}\,\mathbf{SU(2)} \sim \frac{1}{r}S^3$
$\mathbb{H}$-ball
$\mathbb{H} \sim \mathbb{R}^3$





the Heisenberg ball [picture made by Nate Eldredge]

5. Convergence of the Dirichlet heat kernels

Let $p_t^{B_1^{\mathbb{H}}}(\cdot,\cdot)$ be the Dirichlet heat kernel in the unit ball $B_1^{\mathbb{H}}$ of $\mathbb{H}$, and $p_t^{B_r^{SU(2)}}(\cdot,\cdot)$ be the Dirichlet heat kernel in the r-ball $B_r^{SU(2)}$ of $SU(2)$, where the balls are centered at the identity of the groups.

Theorem (Carfagnini, Gordina, T.)

For each $t > 0$

$$\lim_{r\to 0} r^4\, p_{r^2 t}^{B_r^{SU(2)}}\left(\Phi^{-1}\left(\delta_r^{\mathbb{H}}(x)\right), \Phi^{-1}\left(\delta_r^{\mathbb{H}}(x)\right)\right) = p_t^{B_1^{\mathbb{H}}}(x,y). \tag{6}$$

uniformly for $x, y \in B_1^{\mathbb{H}}$.

6. Local convergence of stochastic flows

Let

$$g_s^{B_r^{SU(2)}} := \begin{cases} g_s & s < \tau_{B_r^{SU(2)}} \\ \partial & s \geqslant \tau_{B_r^{SU(2)}} \end{cases} \tag{7}$$

where g_s denotes a hypoelliptic Brownian motion on $\mathbf{SU(2)}$, and

$$\tau_{B_r^{SU(2)}} := \inf \left\{ s > 0, \ g_s \notin B_r^{SU(2)} \right\}. \tag{8}$$

Similarly, let

$$X_s^{B_r^{\mathbb{H}}} := \begin{cases} X_s & s < \tau_{B_r^{\mathbb{H}}} \\ \partial & s \geqslant \tau_{B_r^{\mathbb{H}}} \end{cases} \tag{9}$$

where X_s denotes a hypoelliptic Brownian motion on $\mathbb{H}$, and

$$\tau_{B_r^{\mathbb{H}}} := \inf \left\{ s > 0, \ X_s \notin B_r^{\mathbb{H}} \right\}. \tag{10}$$

Theorem (Carfagnini, Gordina, T.)

For any $0 < r < \frac{1}{7} r_{1/7}$ *there is a continuous stochastic process* Y_s^r *in* $\mathbb{H}$ *such that*

$$Y_s^r :=: \delta^{\mathbb{H}}_{1/r} \Phi \left(g^{B_{3r}^{\mathrm{SU}(2)}}_{r^2 s} \right) \tag{11}$$

in the sense of conditional probability distributions on the event $A_{3r} := \{ s < \tau_{B_{3r}^{\mathbb{H}}} \}$ *and*

$$\lim_{r \to 0} \mathbb{1}_{A_{3r}} \sup_{0 \leqslant s \leqslant T} | Y_s^r - X_s | = 0 \tag{12}$$

in probability.

We use Theorem 3.3.1, page 76, in Kunita 1986 Lectures on stochastic flows and applications, Tata Institute of Fundamental Research Lectures on Mathematics and Physics.

New Frontiers: Layer potentials

$$u(x) = \int_S \rho(y) \frac{\partial}{\partial \nu} P(x, y) d\sigma(y)$$

$$v(x) = G * f = \int_{\mathbb{R}^n} g(x, y) d\mu(y)$$

Riemann-Hilbert and Poincare variational problems

Find a function in $\mathbb{C}$, unanlytic outside of a curve, with prescribed values and jumps on the curve.
Research in progress: Anna Rozanova-Pierrat, Gabriel Claret (CentraleSupélec), Michael Hinz (Bielefeld).

Calssical applications:

- Integrable models, inverse scattering or inverse spectral problem
- the inverse monodromy problem for Painlevé equations
- Orthogonal polynomials, Random matrices
- Combinatorial probability
- Algebraic geometry, Donaldson–Thomas theory

Hilbert transform

$$H(u)(t) = \frac{1}{\pi} p.v. \int_{\mathbb{R}} \frac{u(\tau)}{(t-\tau)} d\tau$$

Research in progress: Anna Rozanova-Pierrat, Gabriel Claret (CentraleSupélec), Michael Hinz (Bielefeld).

Closely connected to the Riemann-Hilbert and Poincare variational problems and is extensively used in analysis and in sygnal processing.

Maxwell and other vector equations

We develop new mathematical tools in the vector case in order to study and solve Maxwell's equations in non-Lipschitz, possibly fractal domains. To that extent, we would like to show here one use of those tools with the time-harmonic Maxwell problem completed with a homogeneous Dirichlet boundary condition, which becomes with our notations:

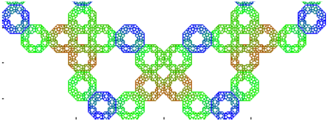
$$\begin{cases} \mathbf{curl}(\mu^{-1}\mathbf{curl}\ \mathbf{E}) - \omega^2\varepsilon\mathbf{E} = \mathbf{f} & \text{on } \Omega \\ \mathbf{Tr}_T(\mathbf{E}) = 0 & \text{on } \partial\Omega \end{cases}$$

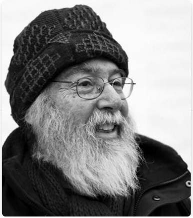
where $\mathbf{f} \in \mathbf{L}^2(\Omega)$ and we look for $\mathbf{E} \in \mathbf{H}(\mathbf{curl}, \Omega)$.
This problem is equivalent to the following variational formulation:
Find $\boldsymbol{E} \in \boldsymbol{H}_0(\boldsymbol{curl}, \Omega)$ *such that* $\forall \boldsymbol{F} \in \boldsymbol{H}_0(\boldsymbol{curl}, \Omega)$*:*

$$(\mu^{-1}\mathbf{curl}\ \mathbf{E}, \mathbf{curl}\ \mathbf{F}) - \omega^2(\varepsilon\mathbf{E}, \mathbf{F}) = (\mathbf{f}, \mathbf{F}).$$

Research in progress: Anna Rozanova-Pierrat (CentraleSupélec), Patrick Ciarlet (ENSTA Paris) et al.

7th Cornell Conference on Analysis, Probability, and Mathematical Physics on Fractals: June 4–8, 2022





In Memory of Professor Robert Strichartz

We will be dedicating the entire conference to Professor Strichartz. A special session will be scheduled during the conference for all to attend and reflect on their thoughts and memories of Bob. Bob is appreciated and recognized for his organizing of the Fractals Conference Community in 2002. He will be profoundly missed by family, friends, colleagues, and most of all, the students he mentored and influenced throughout his career.

A message from the Cornell Department of Mathematics Chair, Tara Holm:

Dear friends,

I am sad to share that our colleague and friend Professor Robert Strichartz died yesterday, 19 December 2021, after a long illness. He was 78.

Thank you for your attention!

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8th Cornell Conference on Analysis, Probability, and Mathematical Physics on Fractals:
June 16–20, 2025

Everybody is invited ! Scientific committee:
Patricia Alonso Ruiz, Michael Hinz, Kasso Okoudjou, Luke Rogers, Laurent Saloff-Coste, Alexander Teplyaev